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THE NOETIC IN MATHEMATICS

Bruno D'Amore

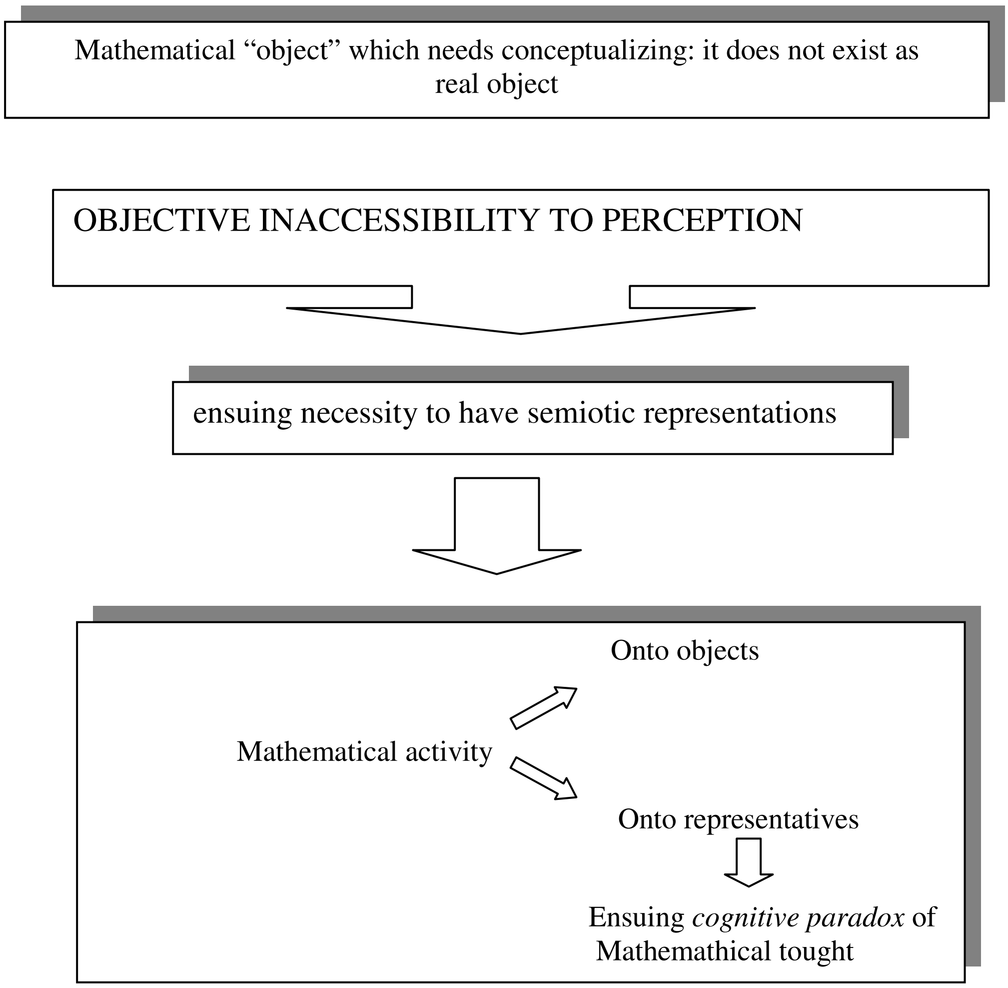
1 Concepts and objects in mathematics

Each and every concept in Mathematics:

1. refers to 'non-objects', what follows from this is that conceptualization is not and cannot be based on meanings resting on tangible reality; in other words, Mathematics does not allow ostensive referrals;
2. is compelled to make use of representations, as there are no 'real objects' that can be shown in stead or as recall; therefore conceptualization, of necessity, has to go through representation patterns. These patterns, for various reasons, and particularly so if they are of a linguistic type cannot be univocal. In this paper 'real object' is meant in the intuitive aspect of 'thing'. The exact meaning is well shown by Aristotle in his *Metaphysics* where he claims that 'a thing' so far as is part of the reality, offers the following features:
    a. it is tridimensional;
    b. it is accessible (approachable) through more than one sense at a time, independent of semiotic representations;
    c. it is possible to separate it materially from other parts of reality, from other 'things';
3. more often than not the Mathematical discourse refers to 'Mathematical objects' rather than Mathematical concepts. This is because objects have become the focus of research more than concepts have (Duval, 1998).

The notion of concept, which most authors consider preliminary or, at any rate, of prime importance, in Duval acquires a secondary place, while what becomes the prime focus is the couple: *sign system - object,* as will be shown here when I refer to the *cognitive paradox of Mathematical thought,* pointed out by Duval himself (1993, 38).

The following outline seems to be more effective than words:



2 The cognitive paradox in mathematical thought

Let us consider this *paradox* as expressed by Duval (Duval, 1993, 38):

> *"(...) on the one hand, learning Mathematical objects can only be a conceptual learning, and, on the other hand, any activity on Mathematical objects is made possible merely by means of semiotic representations. This paradox could become a concrete vicious circle as far as learning is concerned. How would it be possible for learners not to get Mathematical objects mixed up with their own semiotic representations if the one and only relation they have is with semiotic representations? (Learners are bound to confuse Mathematical objects with their semiotic representations because they can relate only to these representations).*

Being unable to build up a direct access to Mathematical objects, which can only happen through a semiotic representation leads to an unavoidable confusion, or nearly unavoidable. And, on the other side, how can learners master Mathematical procedures, if they do not already possess a conceptual learning of the objects represented? This paradox becomes further intriguing if Mathematical activity is identified with conceptual activity and if semiotic representations are seen as secondary or extrinsic".

For a clear definition of terms, without however, any claim at completeness, as these terms are not always used with an identical meaning, I prefer to state the meanings and symbols I will use hereafter:
semiotics $=_{df}$ representation realized by means of signs
noetic $=_{df}$ conceptual acquisition of an object.

From now on:
$r^m =_{df}$ is intended to mean semiotic register (m = 1, 2, 3, ...)
$R^m{}_i(A) =_{df}$ semiotic representation i-nth (i = 1,2, 3, ...) of an object A within the semiotic register r^m.

One may notice that if the semiotic register changes, the semiotic representation will also by necessity change, whilst the opposite is not always true; *i. e.* the semiotic representation may change even when we keep the same semiotic register.

3 Semiotic and noetic in the learning of mathematics

In Mathematics, the conceptual acquisition of any object has to go through the acquisition of one or more semiotic representations (Duval 1988, b, c; 1993-1998; Chevallard, 1991; Codino, Batanero, 1994).
Once more, I will use a graphic to illustrate the issue under consideration, as it results in being more effective and more incisive.

characteristics
cognitive activities
of semiotics

representation
treatment
conversion

these
are all
different

object A to be represented

choice of special features of A

REPRESENTATION R^m_i (A) in a given semiotic register r^m

transformation of representation: TREATMENT

NEW REPRESENTATION ($i \neq j$) R^m_j (A) in the same semiotic register

transformation of register: CONVERSION

NEW REPRESENTATION ($n \neq m$) R^n_h (A) in a *different* semiotic register r^n

(m,n,i,j,h = 1,2,3,...).

I would like to draw the attention to the arrows which, in the first part of the graph, point upwards. Here is why this is so. The distinctive features of the object A depend upon the semiotic ability of representation in the chosen register. If we chose a different register, other features of A would be considered. This is due to the fact that two representations of the same object, though in different registers, will have different contents.

4 Characteristics of noetic

The conceptual acquisition of Mathematical object is based upon two of its 'strong' features (Duval, 1993):

a) the use of more than one register of semiotic representation is typical of the human thought;
b) the creation and development of new semiotic systems is symbol (historical) of the progress of knowledge.

These considerations underline the tight interdependence between noetic and semiotics, as we proceed from one to the other. It is not just that there is no noetic without semiotics, but that semiotics is taken as a necessary feature to allow the first step towards noetic. More in depth information is now needed about the theory that R. Duval has been developing in the past few years. In his theory, a central place is given to conversion as opposed to all other functions, and in particular to treatment, which is considered by most as crucial from the point of view of the process of learning in Mathematics.

5 An attempt at 'defining' construction

The construction of mathematical concepts is, of consequence, tightly dependent on the ability to use *more* registers of semiotic representations of those concepts:

a) to represent them in a given register;
b) to treat such representations within the one and same register;
c) to convert such representation from a given register to a different one.

These three elements taken together and the previous considerations point out the tight link to be found between noetic and constructivism. What we mean by 'construction of knowledge in Mathematics' is in actual facts the combination of these three 'actions' on concepts. We mean by it the very expression of the ability to *represent* concepts (choosing their specific traits); to *treat* the representation thus obtained within a given register; to *convert* these representations from one register to another.

It is as if we were defining the base operations, which, taken together, clarify that 'construction' which, otherwise, will remain a mysterious and ambiguous term, subject to all sorts of interpretation, even a metaphysical one.

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Abstract
D’Amore draws his inspiration from the original discussions of Raymond Duval, and his contribution forms part of the research being done in the NRDM of Bologna University. He attempts to draw out and substantiate the diverse hypotheses that lie at the foundations of unsuccessful devolution, and thus, at the foundations of the schooling of mathematical awareness, exploring topics of research about ‘concepts’ and ‘objects’ in mathematics.

Résumé
D’Amore puise son inspiration dans les discussions originales de Raymond Duval (Strasbourg), et sa contribution constitue une partie importante de la recherche poursuivie au NRDM de l’Université de Bologne. Il tente de décrire et de donner forme aux diverses hypotheses qui composent la base de la devolution infructueuse et donc de l’éducation de la conscience mathématique, explorant des sujets de recherche tels les concepts et les objets en mathématiques.

Samenvatting
D’Amore bouwt voor op het origele werk van R. Duval (Strasbourg). Deze bijdrage sluit aan bij het onderzoek dat in Bologna door de onderzoeksgroep NRDM wordt verricht. Ze gaat in op enkele fundamentele aspecten van de mathematische bewustzijnsontwikkeling.

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